Health Assessment of a PC Box Girder Bridge

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1 INTRODUCTION

Bridge as a structural work is designed to withstand all actions and environmental influences with appropriate degree of reliability and must allow usage for the intended purpose during the whole intended lifetime. This simple definition comprises of three basic aspects that the structures must follow; safety, serviceability and durability. To meet these requirements various design criteria being defined depending on major structural materials used (steel, concrete, timber…), purpose of the structure, environ-mental conditions etc. The state when any design criterion is reached is called the limit state and exceeding this limit leads to a failure. Naturally, consequences of failures are diverse. They range from collapse that directly threatens safety of the people or their property, to defects that affect only the appearance of the structure. States that are associated with collapse are ultimate limit states (ULS). The states dealing with damages that affect comfort of people, appearance, efficient use or durability of the structure are service limit states (SLS). The probability that ULS will be reached has to be very small, the target probability $10^{-4}$ is assumed to be sufficient for ductile mode of failure. While in case of SLS the target probability $10^{-2}$ is supposed to be sufficient.

The rating of damage severity should be based on how much the damage influences safety (safety margin) of the structure. Besides defects that directly threaten structural safety, there are damages that do not hinder proper performance of the bridge under normal traffic conditions but they decrease its safety margin, which means that the structure does not have required reliability. In such cases, possible overloading like traffic congestion may lead to the collapse. There are also some damages associated with durability (SLS) that accelerate the deterioration processes in structural materials and finally may cause serious failure. An example is cracking in prestressed concrete structures exposed to severe weather conditions that may lead to the corrosion of prestressing reinforcement, reduction of steel area and finally rupture, e.g. sudden collapse of Bickton Meadows footbridge in Hampshire, UK in 1967 caused by corrosion of prestressing tendons.

Contrary to the structural, there is an economical view. The rehabilitation or even the replacement of damaged structures is expensive and it may affect traffic and also the people in that large area. Therefore development of diagnostic methods that enable us to detect and determine severity of the damage has an important role in the bridge industry. Furthermore, engineers, who analyze structure, have to be familiar with design criteria and design philosophy, because damages have diverse seriousness regarding the safety and serviceability of the structure.

2 DESCRIPTION OF THE BRIDGE

Kishwaukee River Bridge consists of two independent structures southbound bridge and northbound bridge. The bridges were made by post-tensioning of single-cell precast segments. The deck of the bridge has five spans of lengths 51.8 m + 3 x 76.2 m + 51.8 m (170 ft +3 x 250 ft +170 ft) see fig.1. The overall length of the deck is 334 m (1096 ft) . The deck was built by the balanced cantilever method. Each cantilever consisted of seventeen segments 2150 mm (7'-3 3/8") long and one pier segment 1067 mm (3'-6") long. Cast-in-place closures have a length of 984 mm (3'-2 3/4").

![Fig.1 Longitudinal layout of the Kishwaukee River Bridge](image)

North abutment  pier SB4  pier SB3  pier SB2  pier SB1  South abutment
Each of the two bridges support two 3.66 m (12 ft) wide traffic lanes, two shoulders, curbs and parapets, see fig.2. The total width of the structure is 12.8 m (42 ft). The cross-section of the segments is constant except for the first five segments from the pier, where thickness of the bottom slab changes from 203 mm to 457 mm (8 in - 18 in). The web width of 356 mm (14 in) is constant all over the structure. The maximum depth of the girder is 3550 mm (11'-7\(\frac{3}{4}\) in). Segments have match-cast epoxy joints with one shear key in each web. The nominal compressive strength of concrete was 37.9 MPa (5500 psi).

The decks are entirely prestressed by Dywidag high-strength threaded bars with diameter 32 mm (1\(\frac{1}{4}\) in) located in the top and bottom slabs (no draped tendons are present). In longitudinal direction, the number of bars in the top slab (cantilever bars) ranges from 100 in pier segments to 8 bars in segments #17. The number of bars in bottom slab (continuity bars) ranges from 36 in segments #17 to 6 bars in segments #7. Top slab of each segment is also prestressed by 3 bars in transverse direction. Segments are reinforced by mild steel reinforcement grade 60. Each web contains eight stirrups at 254 mm (10 in) spacing. Stirrups were made from two #7 bars (2\(\phi\)22.2 mm) in the first three segments next to the pier, two #6 bars (2\(\phi\)19.1 mm) in the further three segments and from two #5 bars (2\(\phi\)15.9 mm) in the other segments. Longitudinal reinforcement consists of #4 bars (\(\phi\)12.7 mm) spaced at 254 mm (10 in) at both surfaces.

3 DAMAGES IN THE BRIDGE

The webs of southbound bridge suffer from extensive cracking. Eight segments on either side of each pier are heavily cracked. The crack pattern is very different even within one segment (east and west web). The angle of the cracks varies from 10° to 42°. The widest cracks are very flat sloping at 15° and usually propagating from the bottom part of the female key (crack “A” in fig.3) towards the next segment. In many segments, it can be observed that crack propagates from bottom part of the male key (crack “B” in fig.3). These cracks are usually shorter and less wide than the former one. In most of

![Fig.2 Cross-section of the bridge deck](image)

![Fig.3 Typical crack pattern](image)
widest cracks are located next to the female key having maximum width of 0.75 mm (0.03 in). However in the middle of two segments, it was found to be 0.65 mm (0.026 in). The most frequently observed crack width is 0.40 mm (0.016 in).

The reason for the cracking is known. After completion of the bridge, it was found out that the epoxy glue did not harden properly in most of the joints. The epoxy was not able to carry any shear stresses and instead was acting as a lubricant that caused reduction of friction coefficient in the joints. Substantial part of the shear forces was concentrated at the shear keys. The rest was transferred across the joints by friction mostly in lower part of segments where high compressive axial stresses were present.

Severe cracking was not only a consequence of defective epoxy. Thirteen days after completion of the bridge male shear key of segment SB1-N1 crushed and the inner surface spalled [12]. It resulted in slip of the web at the joint. Relative vertical displacement between the pier and #1 segment was 16 mm (5/8 in).

After failure of SB1-N1 shear key, all defective joints were repaired by steel pins [3]. The smooth contact surface of the joints became indented (toothed) and have substantially improved transfer of shear stresses across the joints but mainly for loads imposed after retrofit (barriers, wearing surface and traffic loads). The steel pins have also enhanced shear resistance of the joints. The lack of performed retrofit was that the structure had not been activated before. Redundant strains and stresses have not been removed and the cracks have not been sealed.

Between years 1983 and 2000 growth of the cracks continued and the width of some cracks have increased twice. An issue of stresses in shear reinforcement has become very important. However, assessment of steel stresses was complex task in the bridge, because many aspects have influenced the flow of internal forces in the webs. For example concentration of shear forces at the shear keys due to defective epoxy, different shear resistance of the joints, retrofit of joints by insertion of the steel pins, creep of concrete. All these aspects do not allow the stresses in reinforcement to be determined by using ordinary models.

Inclined cracks have developed in northbound bridge also. But crack pattern is very different from southbound bridge. Cracks are continuous, usually crossing three segments and slopping at 24°. The most observed crack width is 0.40 mm (0.016 in).

4 TESTING OF THE BRIDGE

Dynamic and static tests were performed on the bridge in 1999 and 2000 with an intention to determine severity of the observed damages.

4.1 Dynamic tests

Three ambient vibration tests were carried out in November 1999, February and May 2000. Dynamic properties were measured on both bridges to assess sensitivity of the technique on this sort of damage. The measured fundamental resonant frequencies and corresponding mode shapes were compared with theoretical ones. For this purpose two FEM-models were built, one from beam elements and the second one from shell elements. Comparison of measured and computed vertical resonant frequencies is tabulated in table 1.

Table 1 lists small differences between resonant frequencies of northbound bridge and southbound bridge. Dynamic properties of both bridges has not changed between years 1986 and 2000 in spite of recorded growth of the cracks in the webs of southbound bridge. Higher frequencies were measured in winter season while the lowest in May. The maximum temperature difference was 19°C (34°F). Those measurements are consistent with an effect of reduced temperature on modulus of elasticity of concrete.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Bridge</th>
<th>1.mode</th>
<th>2.mode</th>
<th>3.mode</th>
<th>4.mode</th>
<th>5.mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured by CTL</td>
<td>southbound</td>
<td>1.61-1.65</td>
<td>2.06-2.08</td>
<td>2.64-2.66</td>
<td>-</td>
<td>3.90-3.98</td>
</tr>
<tr>
<td>Measured by UIC</td>
<td>northbound</td>
<td>1.62-1.69</td>
<td>2.13-2.18</td>
<td>2.75-2.79</td>
<td>-</td>
<td>3.97-4.05</td>
</tr>
<tr>
<td>Computed</td>
<td>not damaged</td>
<td>1.650</td>
<td>2.132</td>
<td>2.767</td>
<td>3.010</td>
<td>4.032</td>
</tr>
</tbody>
</table>

* Measurements made by Construction Technology Laboratory in 1986
Computed frequencies and mode shapes were acquired on non-damaged FEM-models with a modulus of elasticity of concrete $E_{c,\text{dyn}} = 1.07 \times E_{c,\text{stat}}$, concrete density 2450 kg/m$^3$ and 89 mm (3.5 in) thick wearing surface. The comparison of measured and computed first six vertical mode shapes is shown in fig.4.

**4.2 Static load Tests**

Two Static Load Tests were carried out on the southbound bridge in October and November 2000. Two major objectives were followed by tests:

1) Calibration of the FEM-models particularly in relation to the material properties – modulus of elasticity of concrete $E_{c,\text{stat}}$. 
2) Determination of tangential shear stiffness (reduced shear modulus) of the most damaged web, with intention to estimate stress rate in shear reinforcement.

The first test was performed by two three-axles trucks each weighing 72,400 lbs (322 kN). For the second test the weight was increased to 96,050 lbs and 96,900 lbs (422.6 kN 430.5 kN). Four
different positions of the trucks on the bridge were proposed, see fig.5. Positions #1, #2 and #4 induced maximum deflection in the main spans of length 76.2 m (250 ft) and position #3 generated maximum shear in the web SB2-N4-E.

To meet the first objective, the deflections of the main spans were measured for each position of the trucks using two laser digital barcode reading levels TOPCON DM01, stationed on surveying tripod at piers #3 and pier #4. From comparison of measured and computed displacements tangential static modulus of elasticity of concrete was determined $E_{c,est} = 38,000$ MPa (5,515 ksi).

To support assessment, strain gauges were installed on the webs of segments located next to the closures see fig.6. Measured concrete strains were used for calculation of curvature eq. (1) and evaluation of the modulus eq. (2).

### Table 2: Displacements in the middle of spans [mm]

<table>
<thead>
<tr>
<th>Position</th>
<th>Span SB4-SB3</th>
<th>Span SB3-SB2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>measured</td>
<td>computed</td>
</tr>
<tr>
<td>1.</td>
<td>+4.7 / +6.2</td>
<td>+4.7 / +6.3</td>
</tr>
<tr>
<td>2.</td>
<td>-1.7 / -2.2</td>
<td>-1.7 / -2.2</td>
</tr>
<tr>
<td>3.</td>
<td>-0.8 / -0.9</td>
<td>-0.6 / -0.8</td>
</tr>
<tr>
<td>4.</td>
<td>- / +0.5</td>
<td>- / +0.6</td>
</tr>
</tbody>
</table>

first load test / second load test, (+) displacement in gravity direction,

\[ \Delta \chi_{meas} = \left( \Delta e_{top} - \Delta e_{bot} \right) / d_{gs} \]  

(1)

\[ E_c = \Delta M / (\Delta \chi_{meas} I) \]  

(2)

Bending moment $\Delta M$ was analytically determined for known position and weight of the trucks. Moments of inertia of composite uncracked cross-sections $I$ were used. The assessed value of modulus ranged from 35,000 MPa to 40,000 MPa.
The test results showed that the flexural stiffness of the bridge is unchanged and no decompression of the joints appeared. It was also confirmed by diagrams total weight of trucks versus displacement that are linear for both measured spans, see fig.7. The tests showed that the damages have had only small influence on the global performance of the bridge.

![Fig.7 Plot of total load versus displacement](image)

In order to assess tangential shear stiffness and reduced shear modulus, the web SB2-N4-E was instrumented by three LVDT sensors orientated in three different directions, see fig.8. The sensors were installed inside chamber at inner surface of the web. The web is located on the east side of #4 segment (fourth segment from the pier #2, in fig.5) and is deemed to be the most damaged element in the structure.

![Fig.8 LVDT sensors](image)

The tangential stiffness of RC elements can be used as an indicator of steel stresses in reinforcement. Very low value indicates yielding of reinforcement. In assumed web, the reduced tangential shear stiffness was determined by eq. (3):

\[
(GA)_s = \frac{\Delta V_o}{\Delta \gamma_{\text{ul}}} = 7432 \text{ MN}
\]  

Shear force \( \Delta V_o \) in eq. (3) was computed for assumed position of the trucks and corresponding average shear strain \( \Delta \gamma_{\text{ul}} \) was determined from measured average strains \( (\Delta \varepsilon_s, \Delta \varepsilon_t, \Delta \varepsilon_L) \):

\[
\Delta \gamma_{\text{ul}} = \frac{[\Delta \varepsilon_s - \Delta \varepsilon_l \sin^2(\alpha) - \Delta \varepsilon_t \cos^2(\alpha)]}{\cos(\alpha) \sin(\alpha)} = 29.19 \times 10^{-6}
\]  

Shear force \( \Delta V_o \) actually comprises effects of two sectional forces - vertical shear force \( \Delta V_t \) and torsion \( \Delta T \):

\[
\Delta V_w = \Delta V_t / 2 + \Delta T z / (2A_t) = 0.217 \text{ MN}
\]  

Reduced tangential shear modulus was determined by:

\[
G_{\tau} = \frac{\Delta \tau_{ul}}{\Delta \gamma_{\text{ul}}} = 6740 \text{ MPa} \quad (978 \text{ ksi})
\]
Shear stresses in eq. (6) were calculated by eq. (7) with geometric properties for assumed segment:

\[
\Delta \tau_{ul} = \frac{S_{v,\text{max}} \Delta V}{l_c} + \frac{\Delta T}{2 A_k b_{w,\text{eff}}} = 0.197 \text{ MPa (28.6 psi)}
\]

(7)

Reduced stiffness and shear modulus were determined with trucks located in the middle of span #3 (position #2). Though, load position #3 induced the highest shear stresses in the web, the measured displacements could not be used, because transverse bending moment and vertical axial stresses due to trucks standing above the web influenced LVDT sensors measurement, see fig.9.

Assessed stiffness and modulus were compared with a shear stiffness of uncracked web that was determined by:

\[
(GA) = A_u G_u / \kappa = z b_{w,\text{eff}} G_u / \kappa = 15,590 \text{ MN}
\]

and shear modulus by:

\[
G_c = E_c / (2(1 + \nu_c)) = 15,833 \text{ MPa (2298 ksi)}
\]

(8)

(9)

![Cracked plate element](image)

Fig.9.Cracked plate element

It was found out that \((GA) / (GA) = 0.477\) and \(G_u / G_c = 0.426\). From the comparison it can be seen that the reduced tangential shear stiffness of the web was about 45% of the stiffness in uncracked state. It means that reinforcement was working elastically for assumed load stage.

![LVDT sensor data](image)

Fig.10 LVDT sensors data record

However when the trucks left span #3 and shear force dropped to zero, LVDT sensors recorded permanent residual displacements, see fig.10. It indicates some inelastic deformations and
possible yielding of reinforcement. To confirm it further test was proposed with LVDT sensors installed on both sides of the web.

An increment of steel stresses in shear reinforcement was calculated to accomplish the assessment. The web SB2-S4-E is reinforced by stirrups made from two #6 bars (2*19.1 mm) spacing at 254 mm (10 in) with reinforcement ratio \( \rho_{st} = 6.311 \times 10^{-3} \) and in longitudinal direction by two #4 bars (2*12.7 mm) spacing at 254 mm with reinforcement ratio \( \rho_{st} = 2.805 \times 10^{-3} \). Crack pattern consists of parallel inclined cracks sloping at 15°, with average crack width of 0.60 mm, see, fig.8. The stresses were determined using stress transformation equations (10) - (12) and (13),(14),(15).

\[
\Delta \sigma_{cd} = \Delta \sigma_{cl} \cos^2 \theta + \Delta \sigma_{cl} \sin^2 \theta - \Delta \tau_{ct} 2 \cos \theta \sin \theta
\]  
(10)

\[
\Delta \sigma_{cr} = \Delta \sigma_{cl} \sin^2 \theta + \Delta \sigma_{cl} \cos^2 \theta + \Delta \tau_{ct} 2 \cos \theta \sin \theta
\]  
(11)

\[
\Delta \tau_{dr} = \left( \Delta \sigma_{cl} - \Delta \sigma_{cr} \right) \sin \theta \cos \theta + \Delta \tau_{ct} \left( \cos^2 \theta - \sin^2 \theta \right)
\]  
(12)

\[
\Delta \sigma_{l} = \Delta \sigma_{cl} + \rho_{st} A_{st}
\]  
(13)

\[
\Delta \sigma_{t} = \Delta \sigma_{i} + \rho_{st} A_{st}
\]  
(14)

Equations (10)-(12) were simplified assuming following assumptions:

1) Crack does not transfer any tensile stresses \( \Delta \sigma_{cr} = 0 \),
2) An increment of horizontal axial stresses at centroid \( \Delta \sigma_{l} = 0 \) and thus \( \Delta \sigma_{cl} = 0 \),
3) An increment of vertical axial stresses \( \Delta \sigma_{t} \equiv 0 \), and thus \( \Delta \sigma_{ct} \equiv 0 \).

The final solution depends on ability of concrete to transfer shear stresses \( \Delta \tau_{dr} \) across the crack. The experiments show some residual shear strength of concrete in the crack due to aggregate interlocking. The shear strength in the crack is mainly influenced by crack width \( w_{avg} \), tensile strength of concrete and maximum aggregate size. According to [5], the residual shear strength of concrete can be determined by eq. (15). If an average crack width is \( w_{avg} = 0.60 \) mm, maximum aggregate size \( d_{g} = 16 \) mm and nominal strength of concrete 37.9 MPa then:

\[
\tau_{ci} = 0.18 \sqrt{L} / \left[ 0.31 + 24 w_{avg} / (16 + d_{g}) \right] = 1.46 \text{ MPa} \ (211 \text{ psi})
\]  
(15)

If it is assumed that concrete is able to fully transfer shear stresses across the cracks, then shear stresses at interface of the crack \( \Delta \tau_{cl} \) can be determined by eq. (16) with shear stresses \( \Delta \tau_{ct} \) computed by eq. (7).

\[
\Delta \tau_{dr} = \Delta \tau_{ct} \left( \cos^2 \theta - \sin^2 \theta \right) = 0.304 \text{ MPa} \ (44.1 \text{ psi})
\]  
(16)

To meet stress equilibrium in the crack, the eq. (11) had to be transformed to eq. (17)

\[
\left( \Delta \sigma_{l} - \rho_{st} \Delta \sigma_{st} \right) \sin^2 \theta + \left( \Delta \sigma_{t} - \rho_{st} \Delta \sigma_{st} \right) \cos^2 \theta + \Delta \tau_{ct} 2 \cos \theta \sin \theta = 0
\]  
(17)

and smeared stresses in reinforcement had to meet condition

\[
\rho_{st} \Delta \sigma_{st} = \rho_{st} \Delta \sigma_{st}
\]  
(18)

Substituting of eq. (18) in eq. (17) a formula for calculation of unknown steel stresses in shear reinforcement was determined:

\[
\Delta \sigma_{st} = \Delta \tau_{ct} 2 \cos \theta \sin \theta / \rho_{st} = 27.8 \text{ MPa}
\]  
(19)

Since shear stresses in the crack due to permanent actions are actually higher than \( \tau_{ci} \) a zero value of \( \Delta \tau_{dr} \) seems to be more realistic.

In this case the eq. (12) had to be transformed to eq. (20) and two unknown stresses \( \Delta \sigma_{st}, \Delta \sigma_{cl} \) in reinforcement were determined by solving of two linear equations (17) and (20). Computed stress increment in shear reinforcement was \( \Delta \sigma_{ct} = 14.9 \) MPa.
\[
\left[\Delta \sigma_s^t - \rho_s^t \Delta \sigma_{sl}^t - \left(\Delta \sigma_s^t - \rho_s^t \Delta \sigma_{sl}^t\right) \right] \sin \theta \cos \theta + \Delta \tau_{sl} \left(\cos^2 \theta - \sin^2 \theta\right) = 0 \quad (20)
\]

Sectional forces induced by trucks located at the position \#3 were used in the assessment and value of \( \Delta \tau_{sl} = 0.350 \) MPa (50.8 psi) was figured out.

5. CONCLUSIONS

1) The webs of the Kishwaukee Bridge can be deemed as non-prestressed, because prestressing units are located in the slabs. Therefore an existence of the cracks is expected here and less rigorous crack width limits can be applied. However, inadequate growth of the cracks after retrofit has attracted an attention on the stress rate in shear reinforcement. The web cracking as a damage got further dimension in relation to the design criteria. Besides threat of steel corrosion (durability), the higher steel stresses that had developed during construction might threaten shear strength (safety) of the structure.

2) The retrofit has enhanced shear resistance of the joints, which is a fundamental assumption for development of the shear strength of southbound bridge. However we do not know, how existing crack pattern would influence a flow of internal forces due to ultimate load and how higher steel stresses would affect shear strength of the structure.

3) Unfortunately there are no non-destructive methods that allow direct determination of the ultimate strength. Structure in service must not be subjected to ultimate load because it causes such permanent damages in structural materials, that the structure becomes unfit for further service. Testing can be carried out only by a service load (proof testing). On the other hand, the safety design criteria are usually so strict, that a proper performance of the structure under service conditions automatically ensures required safety. However this rule is not applicable in Kishwaukee Bridge.

4) A small sensitivity of both tests (dynamic and static) to detect this sort of damage from global properties of the structure was observed in Kishwaukee Bridge. These defects had little influence on responses like resonant frequencies, corresponding mode shapes and deflections due to static load. Furthermore, material properties of concrete like modulus of elasticity are variable values that depend on many parameters e.g. on ambient conditions e.g. temperature and moisture. So, the deviations of these properties can be larger than the effect of the damage on the global properties of the structure.

5) If damage location is known, the static load test is shown to be a valuable tool for assessment of the damage severity. The performed static load tests indicated that shear reinforcement in the most damaged web was working elastically when a stress increment was ranging from 8 MPa to 15 MPa, because tangential shear stiffness was reduced only to 45% of the value in uncracked state.

6) Higher shear strains accompanied by a transverse bending moment (trucks located at position \#3), caused permanent residual displacements recorded by LVDT sensors. We suppose that the transverse bending moment increased average steel stresses ranging 15 - 28 MPa in shear reinforcement located at exterior surface of the web as the steel could reach the yielding strength and small plastic deformation could occur in some bars. To confirm this theory, further test was proposed with LVDT sensors installed on both sides of the web (inside and outside of chamber).

REFERENCES

NOTATION

- $A_i$ = area enclosed by shear flow;
- $A_{\text{sec}(L)}$ = sectional area of steel bar in longitudinal (transverse) direction;
- $A_w$ = gross area of assumed web's cross-section;
- $b_{w,\text{eff}}$ = effective web width $b_{w,\text{eff}} = b_w / \cos \beta$;
- $d_g$ = aggregate size;
- $d_{gr}$ = vertical distance between strain gauges;
- $E_c$ = modulus of elasticity of concrete;
- $f'_c$ = nominal compressive strength of concrete;
- $G_c$ = shear modulus of concrete;
- $G_t$ = reduced shear modulus;
- $I_c$ = moment of inertia;
- $M$ = bending moment;
- $s_{L(t)}$ = spacing between bars in longitudinal (transverse) direction;
- $S_{c,\text{max}}$ = first moment of area;
- $V_w$ = shear force;
- $w_{\text{avg}}$ = average crack width;
- $Z$ = lever arm;
- $\beta$ = slopping of the web;
- $\epsilon_x$ = axial strain in inclined direction;
- $\epsilon_{z(t)}$ = axial strain longitudinal (transverse) direction;
- $\gamma_U$ = shear strain;
- $\chi$ = curvature;
- $\theta$ = crack inclination;
- $\tau_{dr}$ = shear stresses (d-r axes);
- $\tau_{L(t)}$ = shear stresses (L-t axes);
- $\rho_{sl(t)}$ = reinforcement ratio in longitudinal (transverse) direction;
- $\sigma_{\text{ax}(L)}$ = axial stresses in concrete in longitudinal (transverse) direction;
- $\sigma_{L(t)}$ = axial stresses in RC plate element in longitudinal (transverse) direction;
- $\sigma_{sL(t)}$ = stresses in longitudinal (transverse) reinforcement;
- $\Delta$ = increment, decrement;